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ESTIMATION OF THE FILTRATION PARAMETERS OF A BED BY THE DATA OF NONSTATIONARY INFLOW OF LIQUID TO VERTICAL WELLS

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Numerical algorithms based on the theory of ill-posed problems are suggested for estimation of the filtration parameters of a bed using the results of hydrodynamic studies of vertical wells. These algorithms allow determination of the coefficients of compressibility, efficiency, and hydraulic conductivity of the critical and distant zones and interpretation of the results of the hydrodynamic studies with account for the dependence of the filtration parameters on pressure.

Problems on determination of the filtration parameters of an oil bed by geological-commercial data belong to the class of inverse problems of ground hydromechanics. The distinctive property of them is the dependence of additional data on the possibilities of an industrial experiment. In the present paper, we consider problems on determination of the filtration parameters of porous media by regularization methods. Use of the latter allows one to improve the accuracy and reliability of the filtration parameters under estimation. Results of hydrodynamic studies of vertical wells are used as experimental data.

1. In what follows, we suggest a computational algorithm which makes it possible to estimate the coefficients of compressibility, efficiency, and hydraulic conductivity of the critical and distant zones of the well and values of bed pressure and the radius of the external boundary of the bed.

We estimate the parameters σ, β^*, and p_{bed} by minimization of the functional [1–4]

$$J = \int_0^{t_{\text{exp}}} (\varphi(t) - p(r_{\text{w}}, t))^2 \, dt , \tag{1}$$

when the process of unsteady filtration is described by the equation [5]

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\sigma(r)\frac{\partial p}{\partial r}\right) = H\beta^* \frac{\partial p}{\partial t} , \quad 0 < t \le t_{\text{exp}} , \quad r_{\text{w}} < r < R_{\text{ex.b}} , \tag{2}$$

with initial and boundary conditions

$$p(r, 0) = f(r) , \tag{3}$$

$$2\pi r_{\text{w}} \sigma(r) \left.\frac{\partial p}{\partial r}\right|_{r=r_{\text{w}}} = q(t) , \tag{4}$$

$$p(R_{\text{ex.b}}, t) = p_{\text{bed}} . \tag{5}$$

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Using the method of small perturbations and the condition of stationarity of the Lagrange functional, we can obtain expressions for the gradient of the functional for the case of a uniform bed [1]:

$$J'_{\sigma} = -2\pi \int_0^{t_{\text{exp}}} \int_{r_{\text{w}}}^{R_{\text{ex.b}}} \frac{\partial \psi}{\partial r} \frac{\partial p}{\partial r} \, r dr dt \,, \quad J'_{\beta^*} = -2\pi \int_0^{t_{\text{exp}}} \int_{r_{\text{w}}}^{R_{\text{ex.b}}} \psi \frac{\partial p}{\partial t} \, r dr dt \,, \quad J'_{p_{\text{bed}}} = -2\pi \int_0^{t_{\text{exp}}} \lambda(t)\, dt \,, \tag{6}$$

where $\lambda(t) = \sigma r \frac{\partial \psi}{\partial r}\Big|_{r=R_{\text{ex.b}}}$ and $\psi(r, t)$ is the solution of the corresponding conjugate problem. The process of iteration for recovery of unknown parameters is constructed by a gradient method [1, 5, 6]. The direct problem (2)–(5) and the corresponding conjugate problem are solved by a finite-difference method according to an implicit scheme [7].

In estimating the contamination of the critical zone we assume the bed to be zonally simplex. The coefficients of hydraulic conductivity of the critical and distant zones are found by a gradient method. The components of the gradient of the functional are calculated as follows:

$$J'_{\sigma_1} = -2\pi \int_0^{t_{\text{exp}}} \int_{r_{\text{w}}}^{r_1} \frac{\partial \psi}{\partial r} \frac{\partial p}{\partial r} \, r dr dt \,, \quad J'_{\sigma_2} = -2\pi \int_0^{t_{\text{exp}}} \int_{r_1}^{R_{\text{ex.b}}} \frac{\partial \psi}{\partial r} \frac{\partial p}{\partial r} \, r dr dt \,. \tag{7}$$

Here r_1 is determined from the minimum of functional (1) [6].

The coefficient of efficiency η has the form [8]

$$\eta = 2\pi\sigma \Big/ \ln \frac{R_{\text{ex.b}}}{r_{\text{w}}} \,, \tag{8}$$

where the value of the radius of the external boundary $R_{\text{ex.b}}$ is verified by solving the inverse problem (1)–(5) based on the relation

$$R_{\text{ex.b}} = \sqrt{r_{\text{w}}^2 + \pi \frac{\sigma}{H\beta^*} t_{\text{exp}}} \,. \tag{9}$$

2. The process of development of oil deposits is virtually always accompanied by changes in interstitial pressure and the pressure of uniform compression of rocks. The medium structure changes under the action of external and interstitial pressures. By virtue of this, it is of interest to consider the effect of regimes of development on filtration parameters of the bed.

Experimental studies showed that the dependence of the filtration parameters of the bed on pressure, as a rule, is well approximated by monotonic and convex functions [8, 9]. Therefore, we seek the estimate of the parameter $\sigma(p)$ in this class of functions by the method of descriptive regularization [10, 11]. To do this we solve the variational problem

$$\inf_{\sigma \in D} J(\sigma) \,, \quad J(\sigma) = \int_0^{t_{\text{exp}}} (\varphi(t) - p(r_{\text{w}}, t))^2 \, dt \,, \tag{10}$$

and describe the process of unsteady filtration by the equation

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\sigma(p)\, r \frac{\partial p}{\partial r} \right) = H\beta^* \frac{\partial p}{\partial t} \,, \quad r_{\text{w}} < r < R_{\text{ex.b}} \,, \quad 0 < t \le t_{\text{exp}} \,, \tag{11}$$

with the corresponding initial and boundary conditions of the type (3)–(5). Here D satisfies the following limitations:

$$0 < \sigma_{min} \le \sigma \le \sigma_{max}\,, \quad \sigma_p\,(p) \ge 0\,, \quad \sigma_{pp}\,(p) \ge 0\,, \quad p \in [C_1, C_2]\,, \quad C_1, C_2, \sigma_{min}, \sigma_{max} = \text{const} > 0\,. \tag{12}$$

The expression for the gradient of functional (10) has the form

$$(\nabla J\,(\sigma), \delta\sigma) = -\,2\pi \int_0^{t_{exp}} \int_{r_w}^{R_{ex.b}} \frac{\partial \psi}{\partial r} \frac{\partial p}{\partial r}\, \delta\sigma r dr dt\,, \tag{13}$$

where $p(r, t)$ satisfies conditions (11) and $\psi(r, t)$ is the solution of the conjugate problem

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\sigma\,(p)\, r \frac{\partial \psi}{\partial r} \right) - \frac{\partial \psi}{\partial r} \left(\sigma_p' \frac{\partial p}{\partial r} \right) = H\beta^* \frac{\partial \psi}{\partial t}\,, \tag{14}$$

$$\psi\,(r, 0) = 0\,, \tag{15}$$

$$\pi\sigma\,(p)\, r \left. \frac{\partial \psi}{\partial r} \right|_{r = r_w} = (\varphi\,(t) - p\,(r_w, t))\,, \tag{16}$$

$$\psi\,(R_{ex.b}, t) = 0\,. \tag{17}$$

In numerical realization, variational problem (10) is reduced to the problem of nonlinear programming

$$\min_{\tilde\sigma \in \tilde D} J\,(\tilde\sigma)\,, \quad J\,(\tilde\sigma) = \sum_{j=1}^{M} \tau_j\,(\varphi_j - p_{1j})^2\,. \tag{18}$$

Here $\tilde D$ is the set of grid functions $\tilde\sigma = (\sigma_0, ..., \sigma_k, ..., \sigma_L)$ which are determined at the nodes of the grid $\overline{\omega}_w = \{p_k, C_1 = p_0 < p_1 < ... p_L = C_2, p_k - p_{k-1} = w_k\}$, $\sigma_k = \sigma(p_k)$ and which satisfy the limitations

$$0 < \sigma_{min} \le \sigma_k \le \sigma_{max}\,, \quad k = \overline{0, L}\,; \tag{19}$$

$$(\sigma_{k+1} - \sigma_k) \ge 0\,, \quad k = \overline{0, L-1}\,; \tag{20}$$

$$\left(\frac{\sigma_{k+1} - \sigma_k}{w_{k+1}} - \frac{\sigma_k - \sigma_{k-1}}{w_k} \right) \ge 0\,, \quad k = \overline{1, L-1}\,. \tag{21}$$

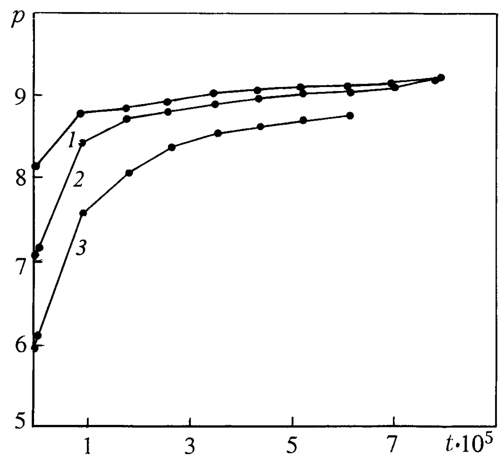
It is assumed that for $p_{ij} \in [p_{k-1}, p_k]$

$$\sigma\,(p_{ij}) = \sigma_{k-1} + \frac{p_{ij} - p_{k-1}}{w_k}\,(\sigma_k - \sigma_{k-1})\,, \quad k = \overline{1, L}\,, \tag{22}$$

where p_{ij} is the solution of the difference analog of problem (11); at the nodes of the grid $\overline{\omega}_h \times \overline{\omega}_\tau$:

$$\overline{\omega}_h = \{u_i, \ln r_w = u_1 < u_2 < ... < u_N = \ln R_{ex.b}, u_i = u_1 + ih, h = (u_N - u_1)/(N-1)\}\,;$$

$$\overline{\omega}_\tau = \{t_j, 0 = t_0 < t_1 < ... < t_M = t_{exp}, t_j - t_{j-1} = \tau_j\}\,, \quad p_{ij} = p\,(u_i, t_j)\,, \quad \varphi_j = \varphi\,(t_j)\,.$$



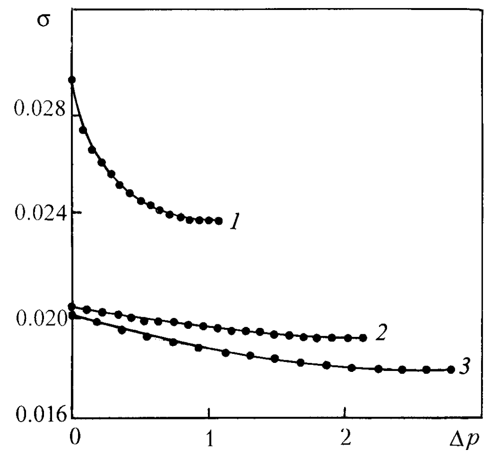


Fig. 1. Curves of pressure recovery taken on well No. 4788.

Fig. 2. Hydraulic conductivity as a function of pressure drop.

TABLE 1. Results of Interpretation of the Curves of Pressure Recovery Taken on Well No. 4788

Number of CPR	Uniform bed					Zonally simplex bed	
	$\sigma \cdot 10^2$	$\beta^* \cdot 10^4$	p_{bed}	$R_{ex.b}$	$\eta \cdot 10^5$	$\sigma_1 \cdot 10^2$	$\sigma_2 \cdot 10^2$
1	2.6	$4.9 \cdot 10^{-4}$	9.2	170	2.4	2.6	2.6
2	1.9	$2.7 \cdot 10^{-4}$	9.2	220	2.2	0.8	2.0
3	1.8	$1.7 \cdot 10^{-4}$	8.9	250	2.2	1.4	1.8

To solve the problem of minimization (18) with limitations (19)–(21) numerically, we use the method of projections of conjugate gradients [10].

The convergence and stability of the algorithm suggested as functions of the form of limitations imposed on the function $\sigma(p)$ to be determined were studied by model problems. An analysis of the results obtained shows that use of *a priori* information leads to improvement of the quality of approximate solutions.

3. Three curves of pressure recovery (CPR) were taken on well No. 4788 (Deposit Shegurchinskoe, Tatarstan) at different instants of time (Fig. 1). Results of the calculation are given in Table 1 and Fig. 2. Half of the mean distance from the studied well to the surrounding operating wells closest to it was taken as initial approximation of $R_{ex.b}$ and the last measurement of pressure — as the initial approximation of p_{bed}. It follows from results of the calculations that contamination of the critical zone is not very high. Interpretation of the first curve indicated that the estimates of hydraulic conductivity of the critical and external zones are of the same order of magnitude. This is connected with the fact that bottom-hole pressure at the initial stage of the experiment was not measured, i.e., the initial portion of the curve, which characterizes the critical zone, is absent.

Estimates of the coefficients of hydraulic conductivity for a uniform bed are in good agreement with estimates of the coefficients of hydraulic conductivity of the external zone [9]. According to the displayed curve, which is constructed based on results of the hydrodynamic studies of well No. 4788, the coefficient of efficiency is $\eta = 2.6 \cdot 10^{-5}$.

Figure 2 presents the dependences of hydraulic conductivity on the pressure drop calculated by three CPR. Results of the interpretation of the hydrodynamic studies by the method of descriptive regularization are in agreement with the calculations given in Table 1.

Results of the calculations show that the suggested technique based on the theory of regularization for interpretation of data of the hydrodynamic studies makes it possible to estimate contamination of the critical zone. The computational algorithm based on the method of descriptive regularization allows recovery of the filtration parameters of the bed with account for their dependence on pressure.

It should be noted that the algorithms suggested above for estimation of the filtration parameters of the bed by the results of hydrodynamic studies of vertical wells make it possible to allow for inflow of liquid after shutdown of the well [5].

NOTATION

r, radius, m; t, time, sec; p, pressure, MPa; $R_{ex.b}$, radius of the external boundary, m; r_w, well radius, m; $q(t)$, well flow rate, m^3/sec; H, bed thickness, m; p_{bed}, pressure in the bed, MPa; $f(r)$, initial distribution of pressure in the bed, MPa; $\varphi(t)$, measurement of bottom-hole pressures in the well during the commercial experiment, MPa; t_{exp}, time of the commercial experiment, sec; σ, coefficient of hydraulic conductivity, $\mu m^2 \cdot m/(mPa \cdot sec)$; $\delta\sigma$, increment of the coefficient of hydraulic conductivity; β^*, coefficient of compressibility, 1/MPa; η, coefficient of efficiency, $m^3/(sec \cdot MPa)$; σ_1 and σ_2, coefficients of hydraulic conductivity of the critical and distant zones, $\mu m^2 \cdot m/(mPa \cdot sec)$; r_1, radius of the critical zone, m; Δp, pressure drop, MPa; J, functional; ψ, Lagrange factor; $\overline{\omega}_h$, $\overline{\omega}_\tau$, difference grid; M, number of time nodes; N, number of spatial nodes; τ, time step; h, step along the space coordinate; $\overline{\omega}_w$, grid of pressure nodes; w_k, step of the pressure grid; L, number of nodes of the pressure grid; C_1 and C_2, minimum and maximum values of pressure; σ_{min} and σ_{max}, minimum and maximum values of hydraulic conductivity; D, set of monotonic and convex functions; $\tilde{\sigma}$, grid function; $\tilde{D}$, set of grid functions. Subscripts: bed, bed; i, j, numbers of nodes of the computational grid; k, numbers of nodes of the pressure grid; ex.b, external boundary; w, well; exp, experiment; min, minimum; max, maximum.

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